PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

On an Electromagnetic Method for the Measurement of the Horizontal Intensity of the Earth's Magnetic Field

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V. *On an Electromagnetic Method for the Measurement of the Horizontal Intensity of the Earth's Magnetic Field.*

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(*From the National Physical Laboratory.*)

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(Plate 3.)

§ 1. *Introductory.*

THE instrument described in this communication is the outcome of a suggestion made by Sir ARTHUR SCHUSTER in 1913. The writer was asked to express his opinion on the practicability of the suggestion and the conditions necessary to secure great accuracy in any measurements which might be made. It appeared that the method proposed would allow a determination of H, the horizontal magnetic intensity, to be made in a few minutes and with a probable error of only a few parts in one hundred thousand. The design of a standard instrument was therefore prepared and submitted to Sir ARTHUR SCHUSTER for approval. Construction of the instrument was commenced in 1914, but owing to the war, it was not completed until late in 1920. In 1919 the writer accepted an appointment with the Admiralty, and as a result, progress with the measurements at Teddington has been slow.

Throughout the work Sir ARTHUR SCHUSTER has rendered most valuable help, and whatever success has been achieved is largely due to him. From the commencement of the investigation the instrument has been called by me the " Schuster Magnetometer."

§ 2. *Previous Measurements.*

WATSON,* in 1902, made the first serious attempt to measure H by an electromagnetic method. In this work, the writer collaborated. A large Helmholtz-Gaugain galvanometer was used. The suspended magnet at the centre, together with fibre, torsion head, etc., formed part of a Kew pattern magnetometer. The results were in good agreement with those obtained with a Kew instrument.

* 'Phil. Trans.,' A, vol. 198, 1902, p. 431.

WATANABE* has recently published an account of an instrument designed for field work. The dimensions of the coils are not measured, the constant of the instrument being found by comparison with that of a standard Helmholtz-Gaugain galvanometer having coils of single turns. The mean diameter of the latter is 20 cm.

S. J. BARNETT,† of the Carnegie Institution of Washington, has made measurements with a Helmholtz-Gaugain galvanometer in which the coils are of bare wire wound in spiral grooves in marble. The mean diameter of the coils is 30 cm. The magnets are circular discs of steel, 2 cm. in diameter, which are polished on both sides in order to serve as mirrors. The probable error of the constant of the instrument appears to be about ± 2 parts in 100,000, and it is stated that the probable error in the value of the current was less than 1 part in 4000. More precise values of the current are needed. The instrument has been used as a sine galvanometer, and the results obtained agree with those given by a Carnegie Institution magnetometer (somewhat similar to a Kew instrument) within about 1 part in 25,000.

§ 3. *Principle of the Schuster Magnetometer.*

The principle of the new magnetometer was first described by Sir ARTHUR SCHUSTER, in 'Terrestrial Magnetism,' March, 1914. Very briefly it is as follows :—

In fig. 1 let AB point accurately to the magnetic north, and let Fi represent the direction and magnitude of the horizontal magnetic intensity produced by a current i through a system of coils, the latter being of such size and arrangement that Fi is practically uniform throughout a sphere having a diameter equal to, or greater than, the length of a small indicator magnet N.S. If Fi is greater than H, the earth's horizontal intensity, and if the component of Fi along AB is in opposition to H, the indicator magnet may, by a rotation of the coil system, be made to set at right angles to AB. H is then determined by

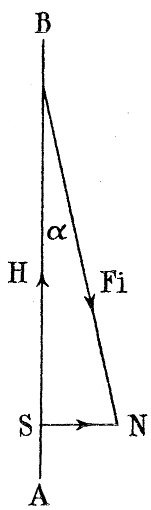

Fig. 1.

$$H = Fi \cos \alpha.$$

If the magnitude of i can be easily adjusted, it may be arranged for α to be very small, when $\cos \alpha$ will be nearly equal to unity, and will vary slowly for comparatively large changes in α.

The direction AB may be most easily determined by reversing the direction of the current (or by turning the coil system through 180 degrees) and then rotating the system until there is no deflection of the indicator magnet when the circuit is either made or broken.

If Fi is less than H and the indicator magnet is caused to set at right angles to Fi instead of to H, then

$$H = Fi/\cos \alpha.$$

* 'Proc. Phys. Math. Soc. Japan,' (3), vol. 2, 1920, p. 210.

† 'Publication 175 of the Carnegie Inst.' of Washington, December, 1921.

The horizontal intensity of the earth's magnetic field in the vertical plane containing F*i* is, then, equal to F*i*, but is in the opposite direction. An instrument of this type may therefore be used to measure the components of H in any direction.

§ 4. *Brief Description of Instrument.*

For constructional purposes, as well as to ensure a uniform magnetic field at and near the centre of the system, a Helmholtz-Gaugain arrangement was adopted (see fig. 2, Plate 3). The equivalent coils on each side of the centre are of 12 turns, of 30 cm. radius, of $\frac{5}{6}$ mm. pitch and of bare copper wire; they are wound on a marble cylinder which, when mounted with axis horizontal, can be rotated about a vertical axis. The magnet at the centre is 1 cm. long and about 5 sq. mm. in cross section; it is supported on a V of aluminium foil by a fine quartz fibre, to which is attached also a reflecting mirror and a damping vane. The damping of the system can be adjusted so that it can be made aperiodic. The magnet is easily removed from its support, and a copper wire of the same weight and of nearly the same dimensions can be substituted. This enables most of the torsion on the fibre to be removed. Reflecting mirrors are attached to the suspended system, to the marble cylinder, and to the case enclosing the magnet. After the magnet has been deflected, the fibre is turned through an equal angle, and no torsion, or change of torsion, in the fibre is therefore possible. The current is determined in absolute measure by means of a current weigher. The leads to and from the magnetometer coils are planned so that the current through them shall have no effect on the magnet. A measurement of H, with calculation complete, occupies (after adjustment of the current) less than four minutes. The total probable error—including error due to uncertainty of the value of the current—is about $\pm$ 4 parts in 100,000.

The horizontal intensity in any direction can be measured, and by a modification of the instrument the vertical intensity can be determined. The modification consists in supporting the coil system with its axis vertical, and spinning a small coil at the centre, the ends of the coil being connected to conductors leading to a vibration galvanometer. For very precise measurements the diameter of the spinning coil, which should consist of a large number of turns, should not exceed 3 cms. The axis of rotation should coincide with the axis of the coil system, and the supporting axis should be a metal rod, cut into two parts connected together by an insulator, the ends of the spinning coil being connected to the two parts of the rod. The circuit would be completed through a vibration galvanometer having a frequency equal to that of the spinning coil. The field due to the Helmholtz-Gaugain coils would oppose the vertical field due to the earth and, when no deflection was obtained with the vibration galvanometer, the two fields would be equal. Careful design of the spinning coil and the vibration galvanometer are needed.

§ 5. *The Marble Cylinder.*

The marble is of fine grain and is from Marmora. It is 61 cm. in diameter and 37 cm. long. Two circular channels, about 2 cm. wide and 0·5 cm. deep, are cut near the ends of the cylinder, and the appropriate twin-screw grooves are cut in these channels to support the wire for the coils. The turning tools used were tipped with bort. Four large circular apertures are made in the cylinder to allow the passage of the main stem and the torsion tube, and to admit light to the mirrors attached to the central system. Two cylindrical holes, parallel to the axis of the cylinder, and at opposite ends of a diameter, permit the introduction of two thermometers, the bulbs of which are centrally situated within the marble.

§ 6. *The Base.*

The base is of gun-metal and consists of a turntable carrying two webs which form a cradle on which the marble cylinder rests. The upper portion of the base, which carries the verniers, rotates about a large central stud, and is supported by a circular race of phosphor-bronze balls. The V ring in which the balls run is in turn supported by a stiff circular spring of phosphor bronze, and it is by adjustment of this spring that a smooth rotational motion is obtained. The turntable is provided with a fine adjustment screw, which operates successfully even when the table is loaded with a weight of 100 kg. There are two verniers 180 degrees apart, and direct readings of angles can be made within 10 seconds. The base was made by Messrs. E. R. Watts & Son, the circular scale being calibrated in the Optical Department of the National Physical Laboratory.

§ 7. *Magnetic Tests.*

When the metal support for the marble cylinder was cast, three lugs were cast with it, and these, together with samples of all parts of the magnetometer, were carefully tested for magnetic quality.

The principal method of testing was similar to that employed for parts of the current balance* and of the Lorenz apparatus. Soft iron wire and ferrous sulphate were used to calibrate the apparatus, and the sensitiveness was sufficient to detect a difference of about one part in 100,000 from unit magnetic permeability. A number of brass rods, tubes and screws were rejected.

In practice, part of the machined surfaces of the webs which carry the cylinder, are in a magnetic field having an intensity of about 2·5, and the copper wire carrying the current is of course in a much stronger magnetic field. These parts, together with various small fittings and parts of the concentric cable, were further tested by bringing them within a few millimetres of one of the poles of a suspended magnet. The pole strength of the latter was 100 C.G.S. units, and the strength of the field in which the

* 'Phil. Trans.,' A, vol. 207, p. 475, 1908.

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specimen was placed was calculated to be about 300. The magnet was provided with a deflecting mirror, and the deflection could be read within about 1 minute of arc. Tested in this way, many metal parts were found to be slightly magnetic, and a considerable number of brass screws were found to be appreciably magnetic.

After completion of the support it was slung so that it could be brought close to a delicate recording magnetograph and rapidly removed again. Had the resultant change in the horizontal intensity near the indicating magnet been as great as two parts in 100,000, it would certainly have been detected. No change was noted.

It is concluded that the permeability of no part of the apparatus differs from unity sufficiently to produce an error of more than one part in 100,000.

§ 8. *The Coils.*

The coils are wound with hard-drawn copper wire in tension, the effective load on the wire being 4 kg. During winding, the cylinder was rotated very slowly, and after about one-third of a revolution the motion of the lathe was stopped for measurements to be made of the diameter of the wire. In all, 64 measurements were made. The mean diameter was found to be 0·563 mm. and the maximum variation from the mean was 0·003 mm. A small cylindrical steel gauge, about the same diameter as the wire, was used as a check on the micrometer.

Each half of the coil system consists of two coils, each of which is 60 cm. in diameter, of six turns and $1\frac{2}{3}$ mm. pitch. The two coils form two interwoven helices, the mean planes of which are coincident, but the commencement and end of one coil are 180 degrees apart from the commencement and end of the other. This twin-coil construction, with bare wires, enables the diametral and axial measurements of the coils to be made with great accuracy, and also enables the insulation resistance between neighbouring turns to be measured with ease.

§ 9. *The Suspended System* (fig. 3, Plate 3).

The magnet M, together with a reflecting mirror m_1, and a damping vane of aluminium foil, are suspended from a fine quartz fibre 25 cm. long. The suspended portions swing in a square case of which two opposite sides are of plate glass.

The upper parts of the other sides of the case are also of plate glass, but the lower portions are of brass and carry the adjusting screws for the damping buffers and the rods with V claws at the ends. These claws serve to support the magnet and copper wire, and enable either of them to be placed in the V support. The parts are shown in fig. 3. The damping buffers have their inner surfaces platinized and are not lacquered; also, to eliminate any electrostatic attraction, a glass tube containing a small quantity of radium bromide is introduced into the chamber.

The top of the case is of brass, and in addition to supporting a tube carrying the torsion head and fibre, it supports four reflecting mirrors m_2, m_3, m_4, m_5, which, after adjustment serve to indicate movements of exactly 90 degrees, 180 degrees, etc. A mirror m_6 attached to a V strip fixed to the marble cylinder can be used to indicate the angle of rotation of the coil system.

In all there are six mirrors, each of 2 metres radius of curvature. One mirror is attached to the magnet system, one to the marble cylinder, and four (90 degrees apart) to the case containing the suspended parts. Three slit sources of light, approximately 90 degrees apart, are used, and images of these are formed on a circular scale of 2 metres radius. Positions exactly 90 degrees apart are obtained by rotating the case and adjusting the four mirrors m_2, m_3, m_4, m_5 in azimuth until the angle between the images is independent of the mirrors used, *i.e.* the axes of the mirrors must be 90 degrees apart. As an alternative method to adjust the mirrors, the large circular scale attached to the turntable can be used.

The magnet case is mounted on a stout tubular metal piece, which rotates about a large metal stud, and is supported by phosphor-bronze balls running in a circular race. The main support is a stiff brass tube passing through an aperture in the marble cylinder, and is secured to the base.

The whole of the suspended system, together with torsion head, fibre, damping buffers, mirrors, etc., can thus be turned rapidly about the central stud. A few seconds enables the angle turned through to be adjusted exactly to 90 degrees, 180 degrees, or any other angle previously arranged for. A fine adjustment screw is provided.

Apart from the marble cylinder and its support, the instrument was constructed by Mr. L. Buxton, of the National Physical Laboratory, and the writer wishes to express his appreciation of the skill of Mr. Buxton in work of this character.

§ 10. *Determination of Constants.*

For a precise knowledge of H it is necessary to have an accurate knowledge of the value of the current in absolute measure and of the constant of the coil system.

The uniformity of the magnetic field, the effect of any torsion on the fibre, the possible magnetic effect of the current in the circuit external to the coils, the magnetic permeability of the coil supports and possible electrostatic effects on the suspended system, are also of considerable importance, but merely demand reasonable care in the design and test of the apparatus and in experimental manipulation.

§ 11. *Precision with which the Current can be Measured.*

The value of the current in absolute measure might be obtained by using the magnetometer coils as part of an absolute electrodynamometer of W. Weber's pattern, or they might be used as part of the fixed coils of a current weigher.

The absolute measurement of the current need not, however, be made with the same, or part of the same, instrument. At the National Physical Laboratory there exists an elaborate current balance, which was constructed in 1908. The balance, as a weighing mechanism, is in perfect order; the insulation resistance between the twin coils has fallen, but not sufficiently to introduce an appreciable error in any measurement of current. Certainly, any possible error is less than a few parts in a million. Unfortunately, it is not possible to re-measure the dimensions of the coils, and it is necessary to assume them to have remained constant.

The balance was overhauled and the coils re-set to be co-axial and co-planar. Subsequently a number of absolute measurements of current were made, a combination of a standard cell and a standard resistance being evaluated. In all, there were twelve determinations, and no single value differed from the mean value by more than 15 parts in a million. The probable error of the value of the current thus determined is believed to be between 3 and 4 parts in 100,000. In the measurements of H with the magnetometer, the current balance was the basis for the absolute measurements of current.

When the current weigher has been provided with new coils, the probable error associated with the value of the current should be reduced to less than $\pm$ 2 parts in 100,000.

§ 12. *Measurement of the Diameters of the Coils.*

The diameters of the coils were measured twice; once along the rake of the helices so that the measurements related to a single coil, and once with the measuring faces of the machine at right angles to the axis of the cylinder. In the latter case, a measured distance is really the addition of the lengths of the radii of the two twin coils. The measurements were made in 24 axial planes at angular distances of 7·5 degrees apart. These planes are numbered 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, 2, $2\frac{1}{4}$ . . . $6\frac{1}{2}$, $6\frac{3}{4}$, in the tables.

The diameters measured were compared with the length of a 60 cm. Hartmann spherical-ended steel gauge. The pressure exerted by the measuring faces on the copper wire of the coils was between 100 and 140 grammes weight.

The results of the first series of measurements of the diameters are given in Table I. In this series, the measurements are related to single coils. The coils on one side of the centre are called A_1 and A_2, and those on the other side are called B_1 and B_2.

In Tables I. and II. the measurements on the coils are summarized in different ways. In Table I. the measurements are arranged to show the departure of the coils from the circular form, whilst the mean values given in Table II. show the taper of the coils.

TABLE I.—First Series (Diameter of Coil—Length of the Hartmann Gauge) at 17° C. Measurements arranged to show Circularity of the Coils. Differences are given in microns (1 micron = 0·001 mm.).

Mean of 5 or 6 measurements.				Plane.
Coil A_1.	Coil A_2.	Coil B_1.	Coil B_2.	
*485·6	482·4	*488·6	476·2	$1\frac{1}{4}$
486·4	*482·6	490·4	*479·2	$1\frac{3}{4}$
*483·8	482·6	*483·8	476·6	$2\frac{1}{4}$
486·9	*479·8	480·2	*473·0	$2\frac{3}{4}$
*484·6	483·6	*474·4	469·1	$3\frac{1}{4}$
490·4	*481·2	470·4	*460·8	$3\frac{3}{4}$
*487·8	485·9	*461·6	457·2	$4\frac{1}{4}$
489·2	*482·2	451·4	*449·6	$4\frac{3}{4}$
*481·2	485·1	*450·6	446·4	$5\frac{1}{4}$
484·6	*477·4	453·6	*445·8	$5\frac{3}{4}$
*476·4	478·1	*464·8	458·7	$6\frac{1}{4}$
483·9	*477·8	481·9	*471·0	$6\frac{3}{4}$
Weighted mean 485·2	481·7	471·0	463·7	
483·4		467·3		
Final Mean = 475·4μ (Coils A and B).				

TABLE II.—Measurements arranged to show Taper of the Coils. Differences in microns.

Mean of 6 measurements.				Turn of wire.
Coil A_1.	Coil A_2.	Coil B_1.	Coil B_2.	
516·1	511·9	464·4	458·7	$\frac{1}{4}$ to $\frac{3}{4}$
511·7	506·4	464·2	455·1	$\frac{3}{4}$,, $1\frac{1}{4}$
503·9	500·7	462·6	452·7	$1\frac{1}{4}$,, $1\frac{3}{4}$
493·7	492·1	460·7	454·1	$1\frac{3}{4}$,, $2\frac{1}{4}$
485·7	482·6	463·9	456·1	$2\frac{1}{4}$,, $2\frac{3}{4}$
474·7	470·4	466·2	459·7	$2\frac{3}{4}$,, $3\frac{1}{4}$
463·7	461·9	472·2	465·9	$3\frac{1}{4}$,, $3\frac{3}{4}$
459·6	458·2	477·4	470·7	$3\frac{3}{4}$,, $4\frac{1}{4}$
466·2	461·1	478·4	474·2	$4\frac{1}{4}$,, $4\frac{3}{4}$
476·4	473·7	483·6	476·6	$4\frac{3}{4}$,, $5\frac{1}{4}$
485·7	479·4	486·4	476·6	$5\frac{1}{4}$,, $5\frac{3}{4}$
Mean 485·2	481·7	470·9	463·7	
Mean A = 483·4		Mean B = 467·3		
Final Mean = 475·4μ (Coils A and B).				

* Mean of 5 measurements.

The detailed results of the second series of measurements are not given here, but they are shown in figs. 4 and 5, which summarize in a graphical form the results of all

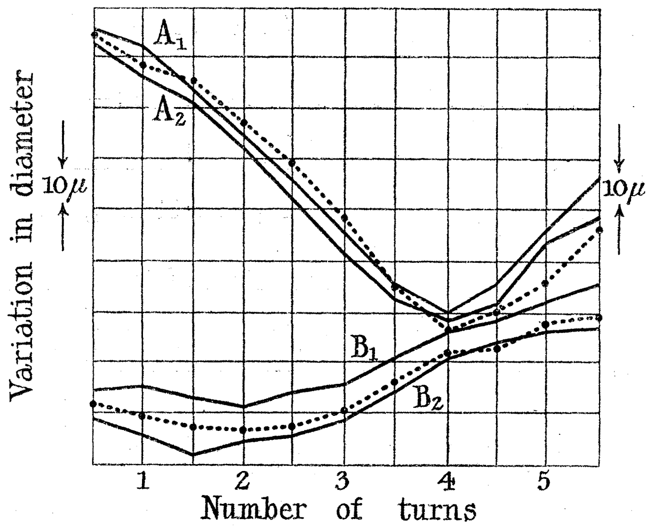

Fig. 4. Showing the taper of the four coils. Length of each coil is 1 cm.

the measurements. Fig. 4 shows the taper of the coils, and fig. 5 shows the departure from the circular form. The dotted lines show the results of the second series of measurements.

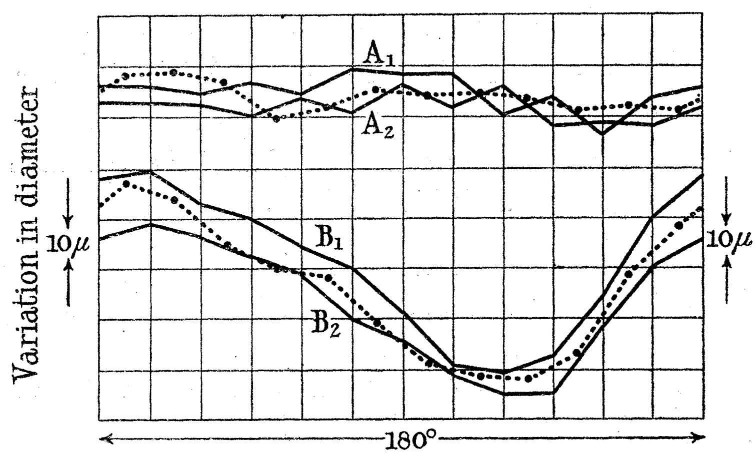

Fig. 5. Showing the departure of the four coils from a circular cross-section.

The mean diameters of the coils at 17° C. measured to the outer limits of the wire are as follows :—

First Series.

A_1 60·0592 } Mean = 60·0591 cm.
A_2 60·0589 }

B_1 60·0578 } Mean = 60·0575 cm.
B_2 60·0571 }

Second Series.

Coils A (A_1 and A_2)	60·0590 cm.	Coils B (B_1 and B_2)	60·0573 cm.

The mean diameters measured to the axis of the wire are :—

A_1	60·0029 cm.	B_1	60·0015 cm.
A_2	60·0026 ,,	B_2	60·0008 ,,

The close agreement between the mean diameters of the four coils bears testimony to the excellent workmanship of Mr. Tribe, who machined the cylinders. The agreement indicates also the degree of precision which can be obtained in this kind of work.

§ 13. *Heating Effect of Current.*

Measurements were made of the diameter of the coils whilst a current of about 0·5 ampere was passing through the coils. For this purpose special agate measuring contact pieces were used in the measuring machine. The two thermometers used in determining the temperature of the marble were placed with their bulbs immediately below the coils. No change in diameter of the coils or in the temperature of the marble was observed during the 30 minutes during which the current was on, or during a subsequent 30 minutes after the current was switched off, other than such as could be accounted for by the change in temperature of the room itself.

§ 14. *Measurement of the Axial Lengths and Pitch of the Coils and the Distance between their Mean Planes.*

The measurements were made on a large pitch-measuring machine, made by Sir W. G. Armstrong Whitworth & Co.* The micrometer microscope attached to the slide of the machine was focussed on the copper wire of the coils, and successive readings were made on each turn. In all, 288 measurements were made along 12 generating lines equally spaced around the cylinder.

The mean lengths of the coils were taken as a basis for calculating the mean pitch, and the mean distance between the pairs of coils was determined by taking the mean of the distances as measured along each generator.

The axial lengths of the coils at 17° C. are as follows :—

$A_1 = 0·9986$ cm.	$B_1 = 0·9990$ cm.
$A_2 = 0·9982$,,	$B_2 = 0·9988$,,

* See "Abs. Measts. of a Resistance by Lorenz Method," 'Phil. Trans.,' A, vol. 214, p. 61, 1914.

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The mean pitch of the coils is 0·083221 cm. Using this figure, the nominal axial reading for any point of a coil can be calculated, and this calculated value, compared with the actual reading of the point, gives the pitch error. Or, symbolically, if p is the mean or nominal pitch and x the reading of the nth wire from the first, the pitch error on this wire is given by $x - np$. The mean pitch errors for each turn are given in fig. 6. Coil A_2 is asymmetrically displaced by a mean amount of 7μ with respect to Coil A_1, and Coil B_2 is likewise displaced with respect to Coil B_1 by 4μ.

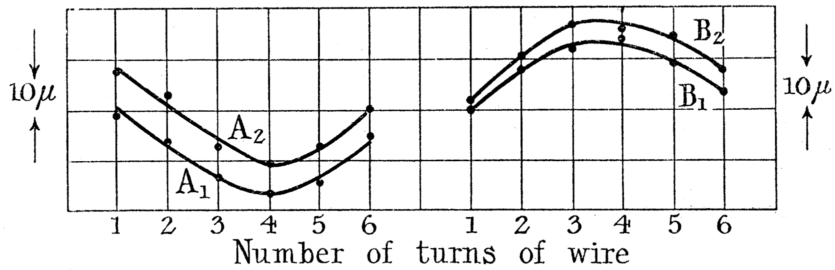

Fig. 6. Showing the pitch errors in the four coils.

The shape of the curves points to a periodicity in the pitch error of about half an inch, which corresponds to the pitch of the lead screw of the lathe on which the cylinder was turned. The same periodicity was found in the coils of the Lorenz apparatus.

The distance between the mean plane of the coils A_1 and A_2 and that of the coils B_1 and B_2 is 30·0098 cm. at 17° C. The irregularities of pitch existing near the ends of the coils were measured and found to be of negligible importance.

The eight ends of the wires of the four coils are bent into the marble as shown in fig. 9. The effect of this is slightly to increase the diameters of the coils, and these particular parts were therefore specially surveyed with a view to applying a correction. However, the calculated correction is 1 part in 60 millions. Similarly, the axial lengths of the coils and the distance between mid-planes must be slightly affected by these irregularities, but again the calculated correction is ridiculously small.

The ends of all the coils were found to lie in an axial plane within less than 0·5 mm.; as the length of wire in the four coils is a little over 45,000 mm., the correction due to any irregularity is less than 1 part in 90,000.

§ 15. *Thermal Coefficient of Expansion of the Marble.*

Observations were made on certain diameters of the coils at temperatures ranging from 12° to 25° C., the whole room being cooled or heated for the measurements. The thermal coefficient of linear expansion was found to be $7{\cdot}9 \times 10^{-6}$ per 1° C. Thermometers can be inserted so that their bulbs are in a central position in the marble.

I am greatly indebted to Mr. S. Attwell and Mr. Johnson, of the Metrology Department, for making the linear measurements.

§ 16. *Calculation of the Constant of the Coil System.*

It is shown later that, for unit current in the circuit, the axial intensity at the centre of a coil system of which each half consists of N turns (the breadth of a coil being comparatively small) is N times that due to unit current in two turns flowing in circles in the mean diametral planes of the coils. The axial intensity at the centre can therefore be calculated from the equation

$$F_x = \frac{4\pi N a^2}{(a^2+l^2)^{3/2}}, \quad \ldots \ldots \ldots \quad (1)$$

where N is the number of turns in half of the complete system, a is the mean radius of the coils, and $2l$ is the distance apart of the mean planes.

In the present investigation,

$$2a = 60\cdot0019 \text{ cm. at } 17^\circ \text{ C.}$$
$$2l = 30\cdot0098 \quad ,, \quad ,, \quad ,,$$
$$N = 12,$$

and the value of F_x calculated by (1) is $3\cdot59595$ cm.$^{-1}$.

Differentiating (1) we get

$$\frac{dF_x}{F_x} = \frac{2l^2-a^2}{a^2+l^2}\cdot\frac{da}{a} - \frac{3l^2}{a^2+l^2}\,\frac{dl}{l} \quad \ldots \ldots \quad (2)$$

or, for $a = 30$ cm. and $l = 15$ cm.,

$$\frac{dF_x}{F_x} = -0\cdot4\,\frac{da}{a} - 0\cdot6\,\frac{dl}{l} \quad \ldots \ldots \ldots \quad (3)$$

Hence, a variation in a of $2\cdot5$ parts in 100,000, or in l of $1\cdot7$ parts in 100,000, produces a change in the value of F_x of 1 part in 100,000. Also, if the cylinder on which the coils are wound expands uniformly with rise of temperature, then $da/a = dl/l$, and the coefficient of change of F_x with temperature is equal to the temperature coefficient of linear expansion of the marble. The coefficient is $+7\cdot9\times10^{-6}$ for a rise of temperature of 1° C. It is believed that the value of F_x is known within 1 part in 100,000.

§ 17. *Uniformity of Field near the Centre of the Coil System.*

It has been assumed, in the calculation of F_x, that the 12 turns constituting each half of the coil system may be regarded as concentrated in the mean planes of the coils. This assumption is justified, because for an axial displacement from the centre equal to half the width (5 mm.) of a coil the change in axial intensity is less than 1 part in a million. In practice, however, the magnet is permanently deflected from the axial position, and it is of importance, therefore, to know if the transverse intensity is also negligible. I have made, therefore, a careful survey of the region in which the indicator magnet may swing.

In the case of two single turns placed co-axial and with their planes parallel, the intensity of the axial magnetic field at the centre for unit current through the turns is

$$F_x = \frac{4\pi a^2}{(a^2+l^2)^{3/2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where a is the radius of a turn and $2l$ is the distance between the planes of the turns.

In the ideal Helmholtz-Gaugain arrangement $l = a/2$, and in such a case the above expression reduces to

$$F_x = \frac{6{\cdot}4\pi}{\sqrt{5}a} = \frac{8{\cdot}99176}{a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

When $a = 30$ cm. and there are 12 turns (supposed to be coincident), $F_x = 3{\cdot}59670_4$ cm.$^{-1}$. The sum of the corrections for $da = 0{\cdot}00095$ cm. and for $dl = 0{\cdot}00490$ cm. is $-0{\cdot}00075_1$. Hence when $2a = 60{\cdot}0019$ and $2l = 30{\cdot}0098$ the value of F_x is $3{\cdot}59595$ cm.$^{-1}$ as obtained by direct calculation from equation (1).

It follows from equation (5) that if in different instruments it is desired to have the same value of axial field and also to use the same value of current, the number of turns in the coils should be directly proportional to the radii. In England, for an axial intensity of about 0·18 C.G.S. unit, a convenient number of turns is 12 for $a = 60$ cm., 10 for $a = 50$ cm., 8 for $a = 40$ cm., and so on. The current in the coils is supposed to be half an ampere.

NAGAOKA has shown* that the axial intensity at any point near the centre can be calculated from the expression

$$F_x = \frac{6{\cdot}4\pi}{a\sqrt{5}}\left\{1 - \frac{18}{125a^4}(8x^4-24x^2y^2+3y^4)\right\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where x and y are the axial and transverse co-ordinates of the point with respect to the centre.

The transverse magnetic intensity is zero at the centre of the system and at all points in the plane midway between the coils. Its value at a point near the centre can be calculated from the expression

$$F_y = \frac{2304\pi}{625a^5\sqrt{5}}\,xy\,(4x^2-3y^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

It follows from (6) and (7) that the ratio of the transverse to the axial intensity can be calculated at any point. This ratio can be calculated with considerable accuracy by means of the equation

$$F_y/F_x = 0{\cdot}576xy\,(4x^2-3y^2)/a^4. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

* 'Phil. Mag.,' vol. 41, 1921.

In a system in which a is 30 cm., the above expression reduces to

$$F_y/F_x = (7{\cdot}1 \times 10^{-7})\, xy\,(4x^2 - 3y^2). \qquad (9)$$

NAGAOKA has given curves showing regions, near the centre of a system, of equal axial intensity and of negligible transverse intensity. He has shown that, throughout a sphere the radius of which is equal to one-tenth of the radius of a coil, and whose centre is coincident with that of the system, the axial intensity is constant within 1 part in 1000. In the present investigation, regions have been examined in which the axial intensity is constant within 1 part in a million, 5 parts in a million, and 10 parts in a million respectively. Regions have also been investigated in which the ratio of the transverse to the axial intensities does not exceed these values. A system of two coils, each coil consisting of 12 turns of 1 mm. pitch, has also been investigated. This system is not quite identical with that of the instrument described in this paper, but very nearly so. It is intended to construct any future systems with coils of 1 mm. pitch.

The results are most conveniently given in the form of curves. The regions in which the axial intensity is constant within 1, 5 and 10 parts in a million, respectively, are shown in fig. 7. The curves given in quadrants A, C, are for a Helmholtz-Gaugain

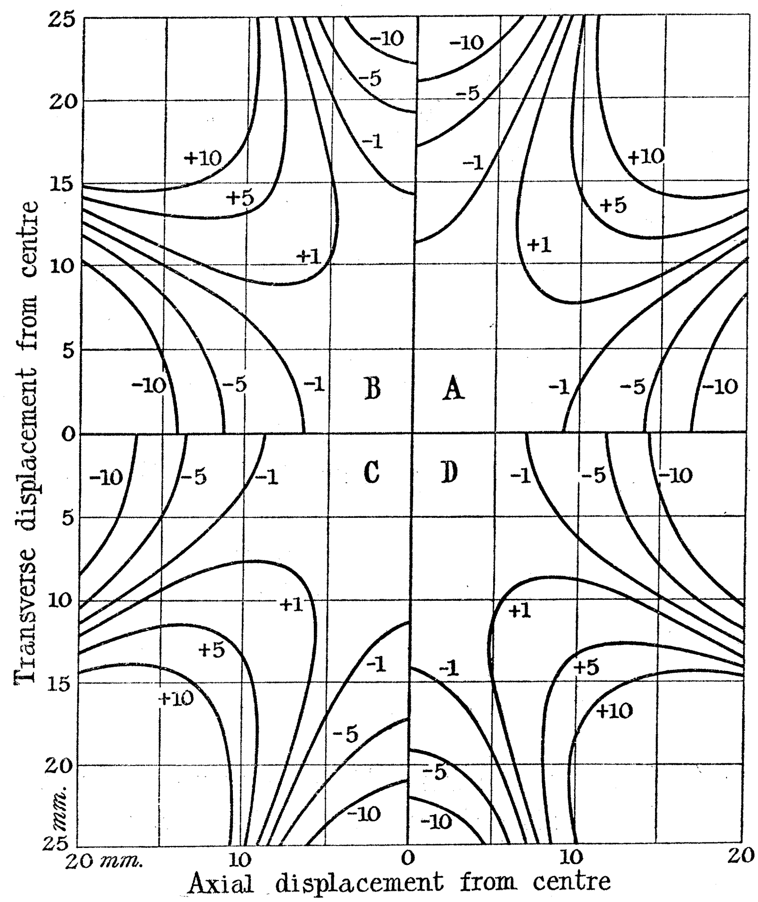

Fig. 7. Showing regions in which the axial intensity is constant within 1, 5, and 10 parts in a million respectively.

system, each coil being of 30 cm. radius and consisting of 1 turn only; those given in the quadrants B, D are for a coil system, each coil being of 12 turns each and of 1 mm. pitch, the distance between the central planes of the coils being equal to the radius of either coil. The + and – signs indicate that the intensity is greater and less, respectively, than at the centre.

It will be observed that one result of using coils of appreciable breadth is slightly to reduce the axial distance over which the axial intensity is practically uniform; on the other hand, the transverse distance is increased. The following table gives the principal results:—

TABLE III.

Variations in axial intensity not exceeding—	Approximate axial distance for system of coils of 1 turn each.	Approximate axial distance for system of coils of 12 turns each.	Approximate transverse distance for system of coils of 1 turn each.	Approximate transverse distance for system of coils of 12 turns each.
	mm.	mm.	mm.	mm.
$\pm 1 \times 10^{-6}$	18	14	23	29
$\pm 5 \times 10^{-6}$	28	25	35	38
$\pm 1 \times 10^{-5}$	34	28	42	44
$\pm 1 \times 10^{-4}$	58	—	74	—
$\pm 1 \times 10^{-3}$	104	—	133	—

The axial and transverse distances for variations in axial intensity of $\pm 1 \times 10^{-4}$ and $\pm 1 \times 10^{-3}$ have been calculated for coils of 1 turn, but are not plotted in fig. 7.

The regions in which the *ratio* of the transverse to the axial intensity is less than 1×10^{-6}, 5×10^{-6}, and 10×10^{-6}, respectively, are shown in fig. 8. The curves shown in quadrants A, A are for a system of coils of 1 turn each and of 30 cm. radius; those in quadrants B, B are for coils of 12 turns each, of 30 cm. radius and of 1 mm. pitch. The + and – signs indicate that the magnetic field is not parallel to the axis; in one case, the direction of the transverse component is from the axis outwards, and in the other case the direction is towards the axis. Inspection shows there is no special advantage of the single-turn system. The following table gives the diameters of spheres throughout which the ratio of the transverse to the axial intensities does not exceed 1, 5 and 10 parts in a million respectively:—

TABLE IV.—Diameters of Spheres with Centres at Centre of System and throughout which the Ratio of the Transverse to the Axial Intensity does not exceed the Values in Column 1.

Ratio of transverse intensity to axial intensity.	Coils of 1 turn.	Coils of 12 turns of 1 mm. pitch.
	mm.	mm.
$1 \cdot 10^{-6}$	22	22
$5 \cdot 10^{-6}$	34	34
$10 \cdot 10^{-6}$	40	42

The differences between the values in Columns 2 and 3 are negligible, and of the same order as the probable errors of the approximate method of calculation used for the system of 12 turns. The curves show that the system with single turns is more advantageous for axial symmetry, but at a slight disadvantage for transverse symmetry.

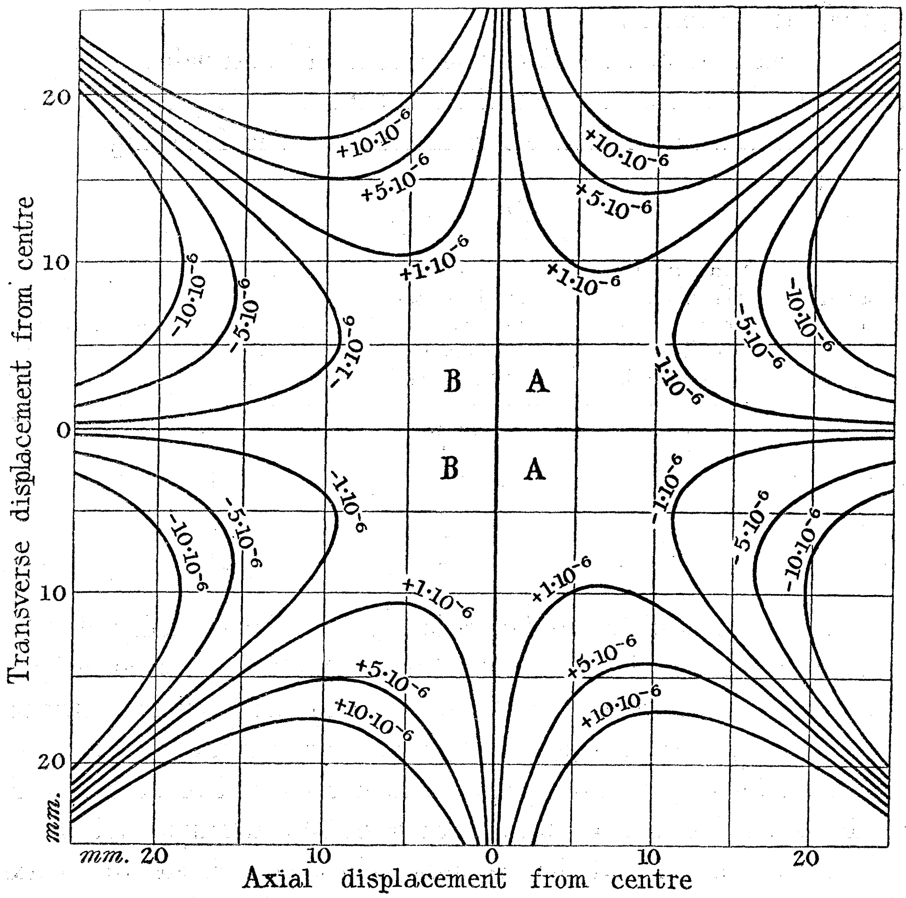

Fig. 8. Showing regions in which the ratio of transverse to axial intensity does not exceed 1, 5, and 10 parts in a million.

The results given in Tables III. and IV. enable one to determine, for any particular measurement, the maximum length of magnetic needle which can usefully be used, and the precision with which its centre must be made to coincide with the centre of the system. This is so obvious that it is not necessary to give any example except that relating to the present investigation. The magnet used was 10 mm. long, and its centre was certainly coincident with the centre of the coil system within 2 mm. From inspection of Tables III. and IV. it is apparent that the error due to the lack of uniformity of the magnetic field produced by the current in the coils, combined with the error due to faulty centering, cannot be greater than about 1 part in a million.

Another valuable feature of the Helmholtz-Gaugain arrangement, whether consisting of single turns or of many, is the ease with which two of them can be compared if their centres can be made nearly to coincide. To construct accurately gauged coil systems is very expensive, and such need only be used as standards.

The marble cylinder I have used will permit the insertion of another coil system 48 cm. in external diameter so arranged that the two systems are co-axial and have a common centre. If this were done and a small magnet were suspended at the centre by means of a reasonably fine quartz fibre, and if torsion were applied to the latter until the axis of the magnet was about 89 degrees to the meridian, the constants of the two systems could be compared by measuring the currents which, when producing fields in opposition, produced no deflection of the magnet system. For a precision of 1 part in 100,000, approximate coincidence only of the centres is necessary. An axial displacement of 1 cm. combined with a transverse displacement of 1 cm. produces an inappreciable error.

The general conclusion drawn with regard to the accuracy of the constant of the coil system used is that the probable error is not greater than 1 part in 100,000.

§ 18. *Connections to Coils.*

The ends of the wire of each coil are bent into the marble as shown in fig. 9. When the cylinder was in the lathe, and after the twin helical grooves had been cut, four cylindrical holes e_1, e_2, e_3, e_4 were drilled to carry the leads of each coil. The holes are

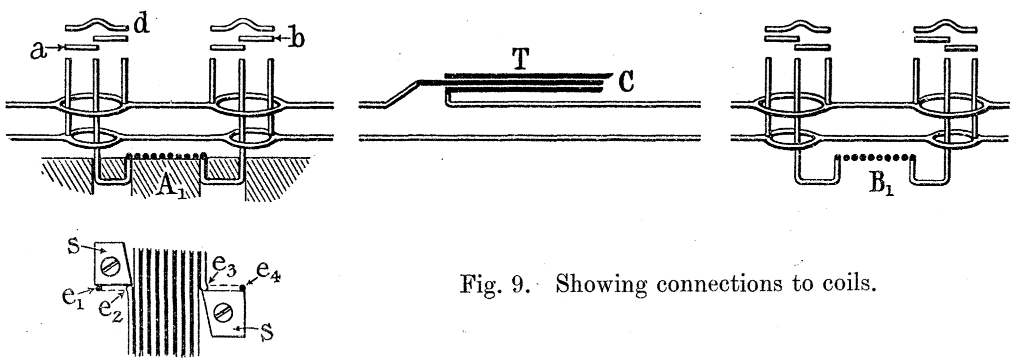

Fig. 9. Showing connections to coils.

slightly greater in diameter than the copper wire, and each hole is about 1 cm. deep. Two pieces of marble of the shape shown by S, fig. 9, were cut from the cylinder and the marble between the holes e_1 and e_2 and between e_3 and e_4 was undercut to allow the passage of the wire. When a coil was commenced, or near completion, the wire under tension was carefully bent into this U-shaped channel, and afterwards a piece of marble was fitted into the hole and screwed into position. This piece served as a key.

The single leads from a coil are therefore, by construction, in a plane containing the axis of the cylinder, and should produce no effect. All four coils begin and end in the same axial plane.

Fig. 9 shows the leads to two of the coils (see fig. 2 also, Plate 3). C is a specially made concentric cable, of which one end passes through guides which are rigidly secured to the cylinder.

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The outer tubular conductor of the concentric terminates in a small copper tube T, and from this, in an axial plane, a stiff copper wire is carried by supports to the coil B. A second stiff wire is soldered to the inner conductor of the concentric, and this passes, in the same axial plane as the other wire, towards the coil A_1. A third wire, in the same axial plane, passes between the two coils. The short radial leads are stiff copper wires supported by insulating pieces, and connection can be made with the concentric cable by bridge pieces such as *a* and *b*. These bridges are short pieces of copper wire, and are soldered in position. A change from the connections when the *a* pieces are used to those when the *b* pieces are employed reverses the current in a coil. If bridge pieces such as *d* are employed, the current flows through the concentric and all other leads to the coils, except the short radial ones. When such connections are made, there should be no axial magnetic field due to the current.

§ 19. *Effect of Leads and the External Circuit.*

To be certain that the current in the leads and external circuit produces no appreciable effect, the following experiment was made :—

The magnet system was supported by a coarser quartz fibre than that employed in the experiments for measuring the horizontal intensity. The moment of the magnet and the diameter of the fibre were such that about 5000 degrees of torsion were required to displace the axis of the magnet 89 degrees from the magnetic meridian.

If ϕ is the angle through which the upper end of the fibre is turned with reference to the lower end, and θ is the angle which the magnetic system makes with the meridian, the angle of deflection $\partial\phi$ for an added axial intensity ∂H is given by

$$\partial\phi = \frac{\phi \cdot \partial H}{H\{1+0\cdot0175\phi \cot \theta\}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In the case under consideration, if H is 0·18000 and ∂H is 1γ the angle $\partial\phi$ is 6 minutes 36 seconds, and the displacement of a reflected spot at 2 metres is 7·7 mm.

Advantage was taken of a "quiet time," *i.e.* when there were no sudden variations in H, to short circuit the leads by the bridge pieces *d*, and thus allow the current to flow through the usual circuit with the exception of the coils. Any deflection on make, break, or reversal of the current was less than 1 mm., and it is concluded, therefore, that the effect of the leads and the whole of the external circuit is negligible. Had the effect been such as to necessitate a correction of 1 part in 100,000, the deflection, on reversal, would have been more than 2 mm.

§ 20. *Effect of Torsion on Fibre.*

It is apparent that, when there is torsion on the fibre, the axis of the magnet cannot be in the magnetic meridian. Further, when the coil system is rotated and the current flows through the coils in such a direction that there is no deflection of the magnet, the axis of the coil system will not be in the meridian. With a twisted fibre, therefore, the axis of the coil cannot be set in the magnetic meridian by the "no deflection" method.

When there is torsion on the fibre and no current in the coils, let θ (fig. 10) be the angle which the axis of the magnet makes with the meridian. If M is the moment of the magnet and H the horizontal intensity, the couple MH $\sin\theta$ is balanced by the torsional couple of the fibre. If T is the torsional couple for 1 degree of twist and ϕ is the actual angle of torsion,

$$\mathrm{MH}\sin\theta = \mathrm{T}\phi. \qquad (11)$$

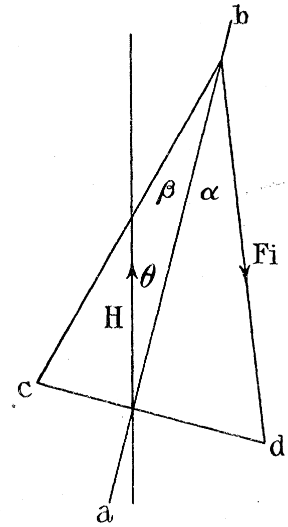


Fig. 10.

When a current i is passed through the coils so as to produce no deflection of the magnet, an axial magnetic intensity Fi is produced which is at an angle θ with the meridian. In practice, therefore, the direction $a\,b$ must, as a first approximation, be taken as being in the meridian.

When making a measurement, the coil system is next moved through a small angle α, and the direction and intensity of the current is changed in order to deflect the magnet through exactly 90 degrees, *i.e.* into the position *cd*. The condition for equilibrium is

$$\mathrm{MH}\cos\theta - \mathrm{T}\phi = \mathrm{F}i\mathrm{M}\cos\alpha. \qquad (12)$$

The magnet is now deflected from its new position through exactly 180 degrees. To produce this effect the coil system is rotated through a relatively small angle $(\alpha+\beta)$. The axis of the magnet is still shown by *cd*, but the poles of the magnet are reversed in position.

The condition for equilibrium is

$$\mathrm{MH}\cos\theta + \mathrm{T}\phi = \mathrm{F}i\mathrm{M}\cos\beta. \qquad (13)$$

From equations (12) and (13) we have

$$\mathrm{H} = \frac{\mathrm{F}i\,(\cos\alpha+\cos\beta)}{2\cos\theta}. \qquad (14)$$

Also

$$2\mathrm{T}\phi = \mathrm{F}i\mathrm{M}\,(\cos\beta-\cos\alpha). \qquad (15)$$

Combining (15) with (11) there results

$$\sin\theta = \frac{\mathrm{F}i}{2\mathrm{H}}\,(\cos\beta-\cos\alpha). \qquad (16)$$

It is apparent that if torsion on the fibre exists, α will not be equal to β, conditionally that H is constant over the interval of time (less than 1 minute) between the measurement of α and β. It is apparent, also, that the torsion on the fibre can be removed, by adjustment of the torsion head, until α is equal to β. However, since in practice Fi is equal to H within less than 1 per cent., the value of θ can be calculated within this limit by the equation

$$\sin\theta = \tfrac{1}{2}(\cos\beta - \cos\alpha) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Knowing θ, an adjustment can be made, if necessary, until its value is negligibly small.

For example, in an early set of observations α was 4 degrees 0 minutes, β was 4 degrees 30 minutes, and θ was calculated to be 0 degrees 1 minute 7 seconds. Experimentally, it was found that 360 degrees of torsion on the fibre produced a deflection of the magnet, from a position near the meridian, of about 2 minutes; hence the torsion on the fibre was about 200 degrees, which could readily be reduced. However, it was not really necessary to do so, for cos 0 degrees 1 minute 17 seconds differs from unity by less than 2 parts in ten millions, and the latter would therefore have been the error in the measurement of H due to 200 degrees of torsion on the fibre.

In general, therefore, after one adjustment, the torsion on the fibre can be neglected. In our experiments, had the torsion been of great importance, the difference $(\alpha-\beta)$ would have been very large indeed. For example, if $\alpha = 2$ degrees and $\beta = 6$ degrees, the value of θ is 8 minutes only, and the error introduced in the calculation of H by neglecting $\cos\theta$ is only 3 parts in one million.

It is concluded, therefore, that any effect of torsion on the fibre cannot, in the present investigation, have produced in any measurement an error so great as 1 part in a million. Some prominence is given to this statement because many experimenters who have seen the magnetometer have expressed surprise at the procedure of suspending a comparatively weak magnet by means of a quartz fibre.

It is important also to note that the first adjustment, *i.e.* adjusting the axis of the coil to be parallel with the equivalent axis of the magnet, need only be a very approximate one. In practice, such adjustment within less than 1 minute of arc is easily made, but if an error of a quarter of a degree is made, and θ is taken in error as zero, the error so introduced is only 1 part in 100,000.

If it is desired to measure the direction of magnetic north, the torsional effect is made negligibly small, and the cylinder turned until there is no deflection of the magnet on making the circuit through the coils. The axis of the coil system then lies in the magnetic meridian. In places where the declination is very constant, great sensitiveness can be obtained by arranging the direction of the current to produce an axial field slightly less than, but opposed to, H.

§ 21. *Arrangement of Circuits and Determination of Intensity of Current.*

The arrangement of the circuit is shown in fig. 11. The current, the value of which is desired, passes through a standard resistance R, and is adjusted until the voltage across the potential terminals of R balances that of the standard cell C. A double commutator T, with copper contacts to reduce thermal e.m.f's., is used to reverse the current through R, and simultaneously reverse the connections to the standard cell. The standard resistance is of strip manganin; it is of nominal resistance 2 ohms, and is divided into halves by a potential lead so as to enable comparisons of each half to be made with a standard 1-ohm coil. The temperature of R is maintained constant at 20·0° C. S is a shunt of known value, and is also immersed in oil. The standard cell is one of a series of "acid" cells; they are kept in an oil bath in a constant temperature room. B is a battery of 50 accumulators. The resistance r consists of about 200 ohms of manganin, and is adjustable in steps of 0·1 ohm, 1 ohm, etc., a sliding short-circuiting

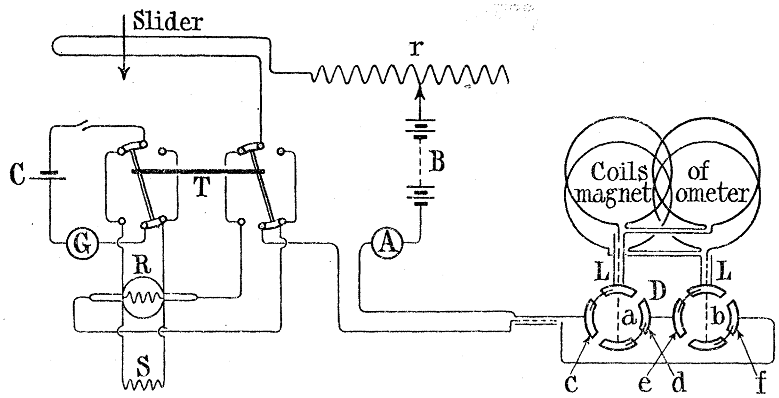


Fig. 11. Circuit.

contact providing the final adjustment. Observations show it is easy to maintain a current constant within less than 1 part in a million.

L L are concentric cables, and lead from a double commutator D to the magnetometer coils. The commutator a reverses the current in two of the coils, one on each side of the centre, and b reverses the current in the other two coils. If the circuit through one half of the system is broken (by placing the cross bar of a turning head in a vertical position) and a conductor placed across c d (or e f) the other half of the magnetometer coils can be used as a complete system. A current of about 1 ampere is then required. Measurements of H, using one half of the system, *i.e.* one coil on each side of the centre, have been made. The results are in complete agreement with those obtained when all four coils are used.

§ 22. *Determination of H.*

After the magnetometer was mounted in position and the cylinder adjusted until its axis was horizontal, the following preliminary experiments were made :—

(1) Determination of effect of leads. The effect was found to be negligible (see § 19).

(2) Adjustment of torsion on fibre. The torsion was made negligibly small (see § 20).

(3) Adjustment of position of damping buffers until the magnet system was nearly aperiodic in magnetic fields, having an intensity of about H/30.

There was no necessity in subsequent determinations to repeat these adjustments, but checks were occasionally made.

The auxiliary current circuit having been completed, the measurement of H comprises the following operations :—

(1) Adjustment of position of the coil system until its axis coincides with the horizontal magnetic axis of the suspended magnet. In this adjustment the axial field due to the coils is approximately in the same direction as the earth's magnetic field. Thirty seconds suffices to make a sufficiently accurate adjustment.

(2) The current through the coils is reversed and the cylinder turned through a few degrees. The torsion tube, etc., are rotated through exactly 90 degrees. The axis of the magnet now tends to set in a direction nearly at right angles to its first position, and is made to do so exactly by adjusting the position of the coil system. Let the angle between the axis of the coil system and the meridian be α.

(3) The coil system is swung through a few degrees with a view to making the resultant field reverse in direction. The torsion tube, etc., are rotated through exactly 180 degrees, and the axis of the magnet now tends to set in a direction approximately 180 degrees away from its position in (2). The reversal is caused to be exactly 180 degrees by adjustment of the position of the coil. Let the angle between the axis of the coil and the meridian be β.

The measurement is now complete, and H is given by

$$H = Fi(\cos\alpha + \cos\beta)/2.$$

F is the constant of the coil system, and i is calculated from the combination value, in absolute measure, of the standard resistance and standard cell.

In practice, the measurements are exceedingly simple and rapidly made. If the current is steady, a complete measurement involving operations (1), (2) and (3) can be conducted within 3 minutes. The following Table gives the results of a few observations made on Friday, July 15, 1921, between 1 and 2 a.m. When making these measurements, adjustment (1) was made only at the commencement and end of the observations. It should be noted that the value of the current was intentionally altered so as to vary the values of i and of α and β. In some measurements, the direction of the current in the circuit was reversed, and the coil swung through 180 degrees approximately, to obtain the proper direction of the axial field.

A sensitive magnetograph for recording the changes in H was in operation simultaneously. A change of 1γ is represented on the magnetograph charts by a displacement of about 2·5 mm., and the time scale is such that 7 mm. represents 1 minute. The temperature was very constant. The changes in H, as recorded by the magnetograph, are given in the last column of the Table; the value at 1 a.m. is that obtained with the magnetometer.

July 15, 1921.

Temperature of cylinder = 20·5° C.

F = 3·59585 cm.$^{-1}$.

Time a.m.	i in C.G.S. units.	α.	β.	$\frac{\cos\alpha+\cos\beta}{2}$.	H = $\frac{Fi(\cos\alpha+\cos\beta)}{2}$.	H as given by Magnetograph.
		° ′	° ′			
1.0	0·051598	5 18	5 12	0·99580	$0{\cdot}18476_1$	0·18476
1.5	0·051598	5 20	5 14	0·99575	$0{\cdot}18475_2$	0·18475
1.11	0·051598	5 26	5 22	0·99556	$0{\cdot}18471_5$	$0{\cdot}18471_5$
1.14	0·051598	5 30	5 24	0·99547	$0{\cdot}18470_0$	$0{\cdot}18470_5$
1.20	0·051428	3 6	2 57	0·99860	$0{\cdot}18466_7$	0·18467
1.24	0·051428	3 2	2 54	0·99866	$0{\cdot}18468_0$	0·18468
1.27	0·051767	7 14	7 10	0·99211	$0{\cdot}18468_0$	$0{\cdot}18467_5$
1.29	0·051767	7 14	7 9	0·99213	$0{\cdot}18468_2$	0·18467
1.32	0·051405	2 32	2 24	0·99907	$0{\cdot}18467_3$	$0{\cdot}18467_5$
1.38	0·051405	2 28	2 22	0·99911	$0{\cdot}18468_0$	0·18468

The dropped figures are for comparison purposes only. It is most satisfactory to note that, with a single exception, the observed and recorded changes agree within the limit (0·5γ) with which the record was read.

As stated in § 21, two coils only, one on each side of the centre, can be used as a complete system. The coils A_1 and B_1 form one system, and A_2 and B_2 the other. On a number of occasions the circuit has been arranged to include two coils only, and excellent agreement between the pairs of coils has always been obtained. The following results were obtained in the early morning of November 26, 1921, when the variations in H were very small. The constant of the instrument when two coils only are used is of course one-half of that when four coils are employed.

F = $3{\cdot}5960_9$ cm.$^{-1}$.						
Time.	Coils.	i.	α.	β.	$\frac{\cos\alpha+\cos\beta}{2}$.	H.
a.m.			° ′	° ′		
1.35	$A_1A_2B_1B_2$	0·51428	3 35	3 33	0·99806	$0{\cdot}18458_1$
1.39	$A_1A_2B_1B_2$	0·51428	3 35	3 35	0·99805	$0{\cdot}18457_9$
1.42	$A_1A_2B_1B_2$	0·51428	3 32	3 33	0·99809	$0{\cdot}18458_7$
1.45	$A_1A_2B_1B_2$	0·51428	3 32	3 35	0·99807	$0{\cdot}18458_3$
F = $1{\cdot}7980_5$ cm.$^{-1}$.						
a.m.			° ′	° ′		
1.53	A_1B_1	1·02858	3 32	3 37	0·99805	$0{\cdot}18458_3$
1.56	A_1B_1	1·02858	3 35	3 38	0·99802	$0{\cdot}18457_8$
2.2	A_2B_2	1·02858	3 32	3 35	0·99807	$0{\cdot}18458_7$
2.6	A_2B_2	1·02858	3 35	3 39	0·99801	$0{\cdot}18457_6$
F = $3{\cdot}5960_9$ cm.$^{-1}$.						
a.m.			° ′	° ′		
2.10	$A_1A_2B_1B_2$	0·51428	3 38	3 40	0·99797	$0{\cdot}18456_6$

The magnetograph showed that from 1.35 a.m. to 2 a.m. H had kept constant within about 1γ. At 2.10 a.m. H was less than at 1.35 a.m. by about 2γ.

On a few occasions photographic records have been obtained, and these enable one to read directly the value of H at any given instant. Simultaneously, the ordinary magnetograph with quartz fibre has been run in order to have two records for comparison. The records thus obtained show conclusively that the current has been maintained steady within 1 part in a million. Changes in H are recorded which last less than 3 seconds.

§ 23. *Comparison with Kew Pattern Magnetometer.*

On March 8 and 9, 1921, a comparison was made of the results obtained with a Kew pattern magnetometer and those obtained with the Schuster magnetometer. Dr. CHREE very kindly made some measurements with a Kew instrument set up a few yards away from the new instrument. During these observations, no measurements were made by the electromagnetic method, but a few measurements were made immediately before and immediately after. A magnetograph was run throughout the whole period. The results obtained are given in the following Table :—

1921.	Time.	Value of H as determined by—		Difference.
		CHREE.	SMITH.	γ
March 8	11.3 to 12.22	0·18488	0·18491	— 3
,, 8	12.26 ,, 13.22	0·18483	0·18496	—13
,, 8	14.44 ,, 16.5	0·18487	0·18488	— 1
,, 8	16.10 ,, 17.6	0·18480	0·18487	— 7
,, 9	10.17 ,, 11.16	0·18460	0·18469	— 9
,, 9	11.32 ,, 12.24	0·18469	0·18471	— 2
			Mean . . .	— 6

Subsequently it was found, on completion of the computation of the P and Q values for the Kew instrument, that the values assigned by Dr. CHREE required a correction, the provisional values being too large by 4γ. The mean observed difference becomes therefore 10γ.

Too much importance must not be attached to this result. The horizontal intensity at Teddington fluctuates rapidly in value owing to the local electric railway and tramway systems ; disturbances of 15γ are not uncommon. For this reason, precision measurements must be made after 1 a.m. However, it appears to be very unlikely that the probable error of observations during the daytime is as great as 10γ. With the new instrument it is possible to follow changes in H which last for only a few seconds.

In the near future, it is hoped to make a second and more complete comparison.

§ 24. *Probable Errors.*

In the absence of artificial disturbances, and natural ones of a rapidly oscillatory character, a convenient value to choose for α, the angle between the axis of the magnetometer and the line pointing to the magnetic north, is 2 degrees. In such case an error of 1 minute in the reading of the angle corresponds to an observational error of 1 part in 100,000.

The probable error in the diametral dimensions of the coils is not greater than 3μ, and the uncertainty in the distance between the mean planes of the two halves of the system may be taken to be of equal magnitude. From equation (3), § 16, the probable error in the constant of the instrument can be calculated by inserting these dimensional errors for da and dl. If the diametral and axial errors are of the same sign, their sum is 8 parts in one million.

The total length of wire in the 4 coils is a little over 45,000 mm. The ends of the coils were determined to be in a line parallel with the axis within less than 0·5 mm., and, when the whole of the circuit external to the coils was completed, the axial intensity due to the current in the circuit was less than 1 part in 200,000 of H. It appears safe to conclude, therefore, that the probable error due to the number of turns being slightly greater, or slightly less, than 24, is not so great as 1 part in 100,000.

The detailed probable errors connected with the coil system are therefore

Diametral	$\pm$ 2	parts in 1,000,000.
Axial	$\pm$ 6	,, ,,
Incomplete turns	$\pm$10	,, ,,

Apart from the magnetic quality of the material, the constant of the coil system may therefore be regarded as known, at the temperature of the measurements, within 1 part in 100,000. This probable error is small, but is in accordance with previous experience with bare wire coils wound on marble.

The magnetic tests lead me to believe that the magnetic permeability of the base, etc., does not differ from unity by more than 1 part in 100,000 and, since the metal is in contact with the marble over only a small part of its surface, the probable error due to the base cannot exceed a few parts in a million. Copper wire is often slightly troublesome owing to being drawn through steel dies. No measurable magnetic effect was obtained with the wire used for the coils, but in future work hard-drawn silver wire, drawn through diamond dies, will probably be used.

The errors due to electrostatic effects, to torsional effects on the fibre, and to changes in the magnet, are considered to be not greater than a few parts in a million. It may be that the magnetic axis of the magnet in the earth's field does not exactly coincide with the axis in a magnetic field having an intensity of 0·006 C.G.S. unit or less, and in certain kinds of measurements errors may be thus introduced. In our measurements, however, the only important angular measurements are those when the magnet is turned through 180 degrees, and in these the field intensities are equal.

The remaining source of error is due to an uncertainty in the value of the current. When the current balance was erected at Teddington the probable error of a determination of current in absolute measure was ± 2 parts in 100,000. Since then the insulation resistance of the coils has fallen considerably, and the dimensions may have altered slightly. The probable error may now be as great ± 4 parts in 100,000.

Finally, we conclude that the probable error of our measurements of H is of the same order as that associated with the current, that is

$$\pm 4 \text{ parts in } 100{,}000.$$

When the coils of the current weigher have been re-wound, the probable error of measurements of H should be reduced to ± 2 parts in 100,000.

§ 25. *Conclusions.*

From the measurements recorded in the previous sections, it is concluded that the type of instrument originally suggested by Sir ARTHUR SCHUSTER can be used to measure the horizontal intensity of the earth's magnetic field with a probable error of the same order as that with which current can be measured. At present this probable error is about ± 4 parts in 100,000, but within a few years it should be reduced to 1 or 2 parts in 100,000. The time occupied in making a measurement and calculating the result is a few minutes only.

My most hearty thanks are tendered to Sir ARTHUR SCHUSTER, F.R.S., to Sir RICHARD GLAZEBROOK, K.C.B., F.R.S., who was Director of the National Physical Laboratory when the magnetometer was commenced, and to Sir JOSEPH PETAVEL, K.B.E., F.R.S., the present Director of the Laboratory. I am indebted also to Mr. S. WATTS, an Observer in the Electrical Department, for much help in making the observations and for aid in the determinations of current intensity.

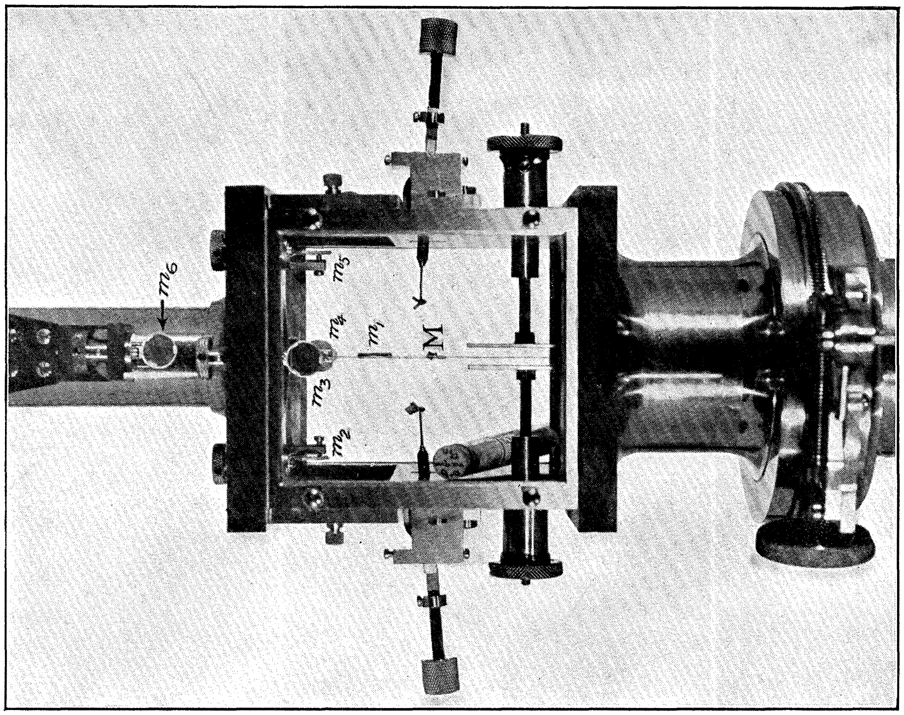


Fig. 3.

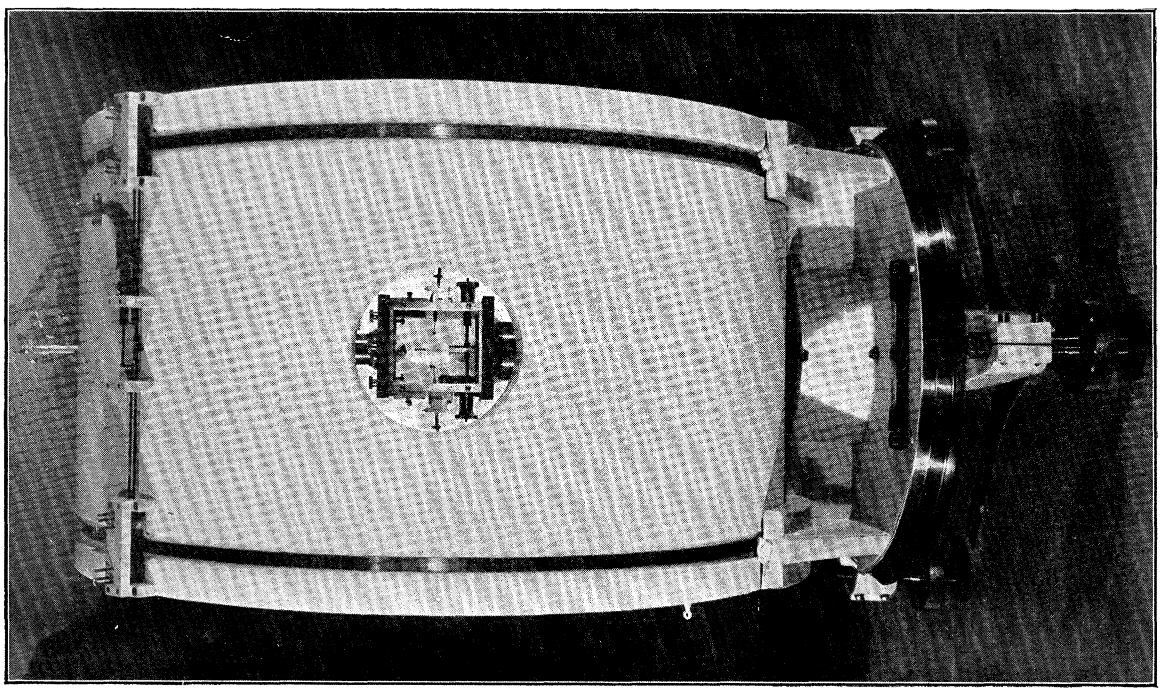


Fig. 2.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

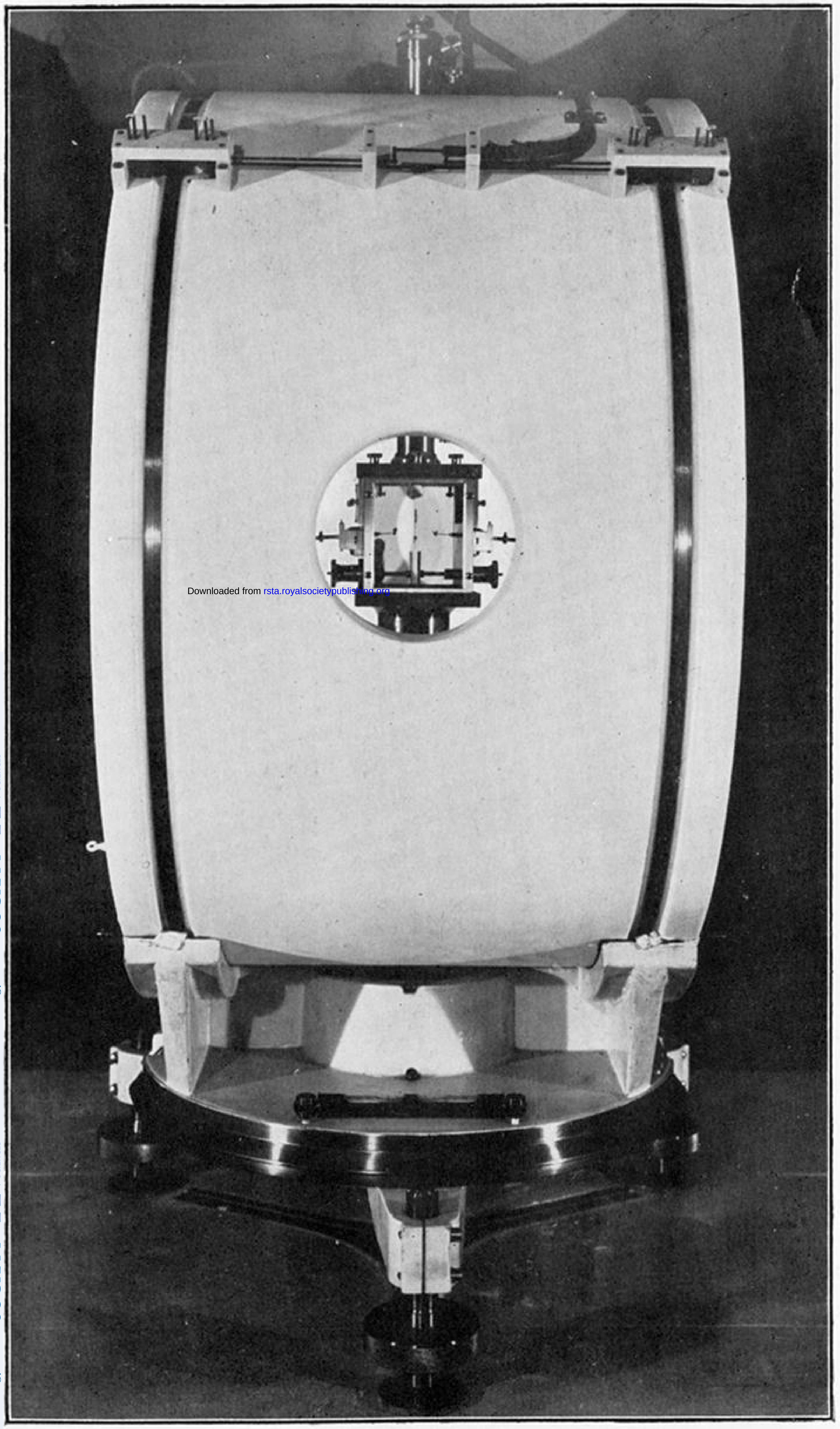
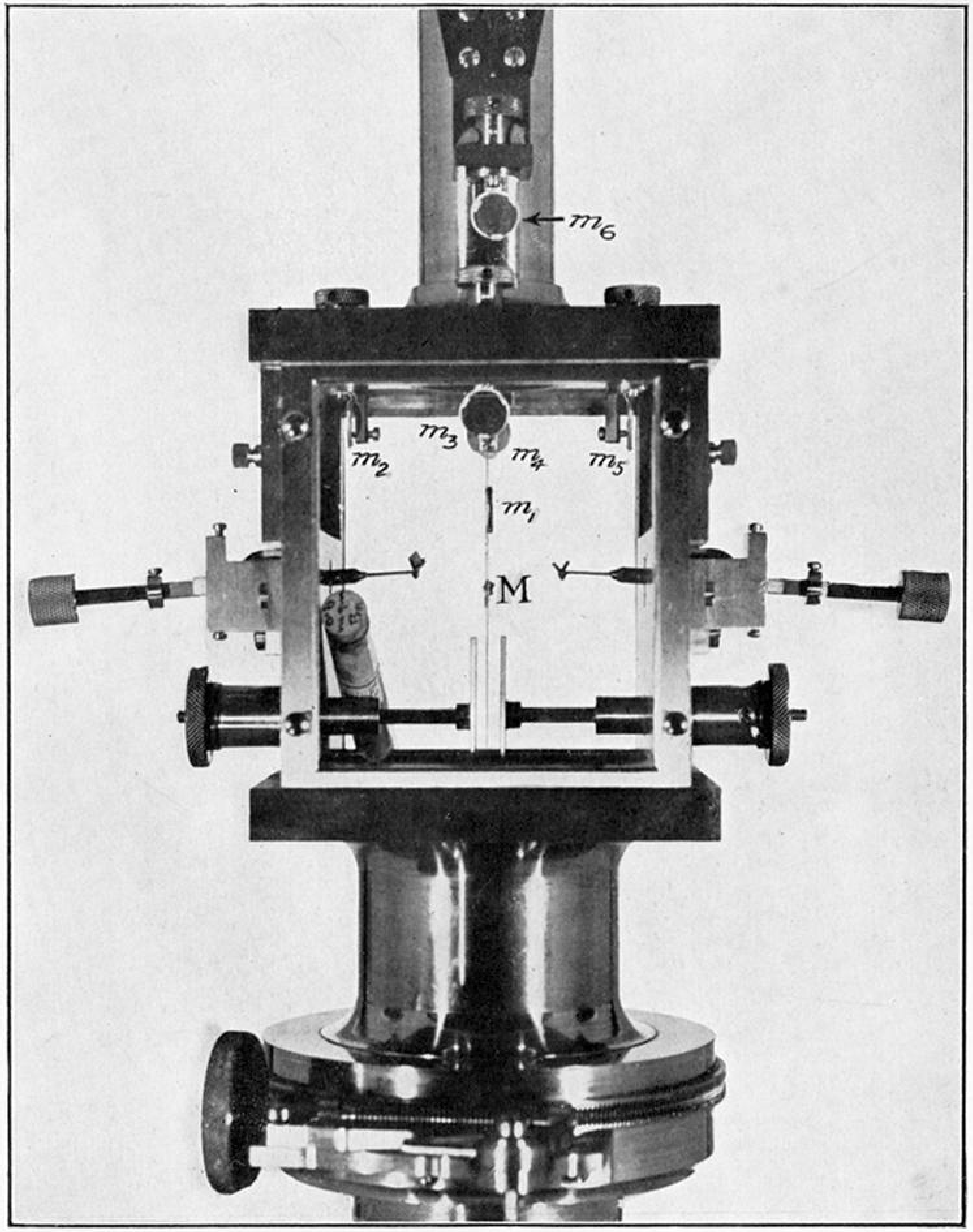


Fig. 2.



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